Supplementary Material for "Liberty, Security, and Accountability: The Rise and Fall of Illiberal Democracies"

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We prove that, for any given set of parameters, our model has an *essentially unique* equilibrium (i.e., Lemma 5 in the main text).

Throughout we use the following conventions. We refer to Proposition Proposition n or Lemma n in the main text as Proposition n or Lemma n. We use Lemma n' to distinguish results that are specific to this supplementary material. The one exception where the naming conventions coincide is for Lemma 5. Unless otherwise stated, equation references will always refer to equations in the supplementary material.

Formally, we say that two equilibria are *essentially equivalent* if, for any t and any g_{t-1}, the probability that an illiberal government is elected is equal in each equilibrium (and hence the voter's and illiberal government's expected payoffs are also equal).[1] For any given set of parameters, we say that the equilibrium is *essentially unique* if all equilibria are essentially equivalent.

Lemma 5 (Essentially unique equilibrium.) *For any given set of parameters, there is an essentially unique equilibrium.*

We begin by identifying three cases that can arise under any censorship policy, c_t:

(I) the voter chooses $g_t = i$ with probability zero;

(II) the voter chooses $g_t = i$ with probability one; or

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[1] In addition, by Lemma 3, if an illiberal government chooses a partial censorship policy, then it is unique across all equilibria.

(III) the voter chooses $g_t = i$ with positive but non-unit probability.

On the equilibrium path, the censorship policy is either $c_t = c_\ell$ (when $g_{t-1} = \ell$) or $c_t = c^*$ (when $g_{t-1} = i$). Therefore, there are at most 9 "types" of equilibria that are characterized by the dot points above. We will refer to an equilibrium where Case I applies when $c_t = c_\ell$ and Case II applies when $c_t = c^*$ as an I-II equilibrium (and similarly for the other 8 equilibria). Lemma 1′ rules out many of these types of equilibria and shows that just four different types of equilibria can exist: an I-I, II-II, III-II, or III-III equilibrium.

Lemma 1′

1. *An I-II and I-III equilibrium does not exist.*
2. *An II-I and II-III equilibrium does not exist.*
3. *An III-I equilibrium does not exist.*

Proof. Part 1. Suppose that Case I applies whenever $c_t = c_\ell$. By Lemma 1, this implies

$$\mu_t^*(1, c_\ell)S < L + \delta A(\pi, q, L, S, \delta \mid \sigma^*). \tag{1}$$

We now show that, under c^*, neither Case II nor III can occur. For any censorship policy c_t and any message m that is sent with positive probability under c_t

$$\mu_t^*(0, c_\ell) \leq \mu_t^*(m, c_t) \leq \mu_t^*(1, c_\ell), \tag{2}$$

therefore, by (1), for any m_t that is sent with positive probability under c^*

$$\mu_t^*(m_t, c^*)S < L + \delta A(\pi, q, L, S, \delta \mid \sigma^*).$$

Thus, the voter chooses $g_t = i$ with probability zero.

Part 2. Suppose that Case II applies whenever $c_t = c_\ell$. By Lemma 1, this implies

$$\mu_t^*(0, c_\ell)S \geq L + \delta A(\pi, q, L, S, \delta \mid \sigma^*). \tag{3}$$

We now show that, under c^*, neither Case I nor III can occur. By (2) and (3), for any m_t that is sent with positive probability under c^*

$$\mu_t^*(m_t, c^*)S \geq L + \delta A(\pi, q, L, S, \delta \mid \sigma^*).$$

Thus, the voter chooses $g_t = i$ with probability one.

Part 3. Suppose that Case III applies whenever $c_t = c_\ell$. By Lemma 1, this implies that

$$\mu_t^*(0, c_\ell)S < L + \delta A(\pi, q, L, S, \delta \mid \sigma^*) \leq \mu_t^*(1, c_\ell)S.$$

Under c^*, Case I cannot occur because if the illiberal government were elected with probability zero when $c_t = c^*$, then c^* must not be optimal—a contradiction. ■

Lemma 2′ says that any equilibrium must be essentially unique.

Lemma 2′ *For a given set of parameters,*

1. *if an I-I equilibrium exists, then it is essentially unique;*
2. *if an II-II equilibrium exists, then it is essentially unique;*
3. *if an III-II equilibrium exists, then it is essentially unique; and*
4. *if an III-III equilibrium exists, then it is essentially unique.*

Proof. *Part 1.* Let σ^* be a I-I equilibrium. Then $A(\pi, q, L, S, \delta \mid \sigma^*) = 0$ and, by Lemma 1 and (2), this implies that

$$\mu_t^*(1, c_\ell)S < L. \tag{4}$$

Now let σ' be another equilibrium. Clearly, if σ' is an I-I equilibrium, then σ^* and σ' are essentially equivalent. For sake of a contradiction, suppose that σ' is not an I-I equilibrium. By Lemma 1′, this implies that σ' is either an II-II, III-II, or III-III equilibrium. In either case, by (2), the voter elects the illiberal government when $m_t = 1$ and $c_t = c_\ell$, i.e.,

$$\mu_t^*(1, c_\ell)S \geq L + \delta A(\pi, q, L, S, \delta \mid \sigma') \geq L, \tag{5}$$

where the last inequality follows from (Lemma 4). But (5) contradicts (4).

Part 2. Let σ^* be a II-II equilibrium. Then $A(\pi, q, L, S, \delta \mid \sigma^*) = 0$ and, by Lemma 1 and (2), this implies that

$$\mu_t^*(0, c_\ell)S \geq L. \tag{6}$$

Now let σ' be another equilibrium. Clearly, if σ' is an II-II equilibrium, then σ^* and σ' are essentially equivalent. For sake of a contradiction, suppose that σ' is not an II-II equilibrium. By Lemma 1′, this implies that σ' is either a I-I, III-II, or III-III equilibrium. However, by Part 1 of this Lemma, σ' cannot be a I-I equilibrium. Therefore, σ' must be an III-II or III-III equilibrium.

First, suppose that σ' is an III-II equilibrium. In this case, the accountability cost of illiberalism is

$$A(\pi, q, L, S, \delta \mid \sigma') = \frac{\Pr[s(\theta_t) = 0]}{1 - \delta \Pr[s(\theta_t) = 0]} \big(L - \Pr[\theta_t = 1 \mid s(\theta_t) = 0] S\big). \tag{7}$$

This follows because if the voter elects the illiberal government just once, then the illiberal government remains in power forever and, under $c_t = c_\ell$, the voter must choose $g_t = i$ if and only if $m_t = 1$. However, because $A(\pi, q, L, S, \delta \mid \sigma') \geq 0$ (Lemma 4), (7) implies that $L \geq \Pr[\theta_t = 1 \mid s(\theta_t) = 0]S$. If this inequality is strict, then we achieve a contradiction because

$$L > \Pr[\theta_t = 1 \mid s(\theta_t) = 0]S \equiv \mu_t^*(0, c_\ell)S \geq L,$$

where the final inequality follows from (6). Otherwise, i.e., if $L = \Pr[\theta_t = 1 \mid s(\theta_t) = 0]S$, then $A(\pi, q, L, S, \delta \mid \sigma') = 0$ and we achieve a contradiction: under σ' and $c_t = c_\ell$, the voter elects the liberal government when $m_t = 0$ and so

$$\mu_t^*(0, c_\ell)S < L + \delta A(\pi, q, L, S, \delta \mid \sigma') = L \leq \mu_t^*(0, c_\ell),$$

where the final inequality follows by (6).

Second, suppose that σ' is an III-III equilibrium. In this case, the accountability cost of illiberalism is

$$A(\pi, q, L, S, \delta \mid \sigma') = \frac{c'(0) \Pr[s(\theta_t) = 0] \Big(L - \Pr[\theta_t = 1 \mid s(\theta_t) = 0] S\Big)}{1 - c'(0)\delta \Pr[s(\theta_t) = 0]}. \tag{8}$$

This follows because (i) if the voter elects the illiberal government with positive but non-unit probability when $c_t \in \{c_\ell, c'\}$, then she must choose $g_t = i$ if and only if $m_t = 1$ when $c_t \in \{c_\ell, c'\}$; and (ii) by Lemma 3, the illiberal government's equilibrium choice of censorship policy, c', will be such that when $m_t = 1$ the voter is indifferent between choosing $g_t = i$ or $g_t = \ell$. However, because $A(\pi, q, L, S, \delta \mid \sigma') \geq 0$ (Lemma 4), (8) implies that $L \geq \Pr[\theta_t = 1 \mid s(\theta_t) = 0]S$. As shown in the previous step (when assuming that σ' is an III-II equilibrium), this leads to a contradiction with (6).

<u>*Part 3.*</u> Let σ^* be a III-II equilibrium. Then $A(\pi, q, L, S, \delta \mid \sigma^*) = \bar{A}(\pi, q, L, S, \delta)$ (see Proof of Point 2 of Proposition 3 in Appendix A) and, by Lemma 1 and (2), this implies

that for any message m_t that is sent with positive probability when $c_t = c^*$ we have

$$\mu_t^*(m_t, c^*)S \geq L + \delta\bar{A}(\pi, q, L, S, \delta). \tag{9}$$

Now let σ' be another equilibrium. If σ' is a III-II equilibrium, then σ^* and σ' are essentially equivalent because, when $g_{t-1} = i$ (and hence $c_t = c^*$), the illiberal government is elected with probability one and, when $g_{t-1} = \ell$ (and hence $c_t = c_\ell$), the illiberal government is elected with probability $\Pr[s(\theta_t) = 1]$. For sake of a contradiction, suppose that σ' is not a III-II equilibrium. By Lemma 1′, and Parts 1 and 2 of this Lemma, σ' must be a III-III equilibrium. Therefore, by Lemma 3, the illiberal government's equilibrium choice of censorship policy c' is such that both messages $m_t = 0$ and $m_t = 1$ are sent with positive probability, and

$$\mu_t^*(0, c')S < L + \delta A(\pi, q, L, S, \delta \mid \sigma') = \mu_t^*(1, c').$$

Notice that the illiberal government cannot be indifferent between all censorship policies: if they were, the full censorship policy, c_F, would lead to a non-deterministic election outcome—an impossibility because c_F induces a single belief for the voter and the voter breaks ties in favor of the illiberal government. Therefore, by Lemma 4, $A(\pi, q, L, S, \delta \mid \sigma') \leq \bar{A}(\pi, q, L, S, \delta)$. But this is a contradiction: c' cannot be optimal for the illiberal government since they could guarantee their reelection by deviating to the policy $c_t = c^*$.

Part 4. Let σ^* be a III-III equilibrium, and let σ' be another equilibrium. By Lemma 1′ and Parts 1–3 of this Lemma, σ' must be a III-III equilibrium. Note that, by (2), in both σ^* and σ' the illiberal government's probability of election when $g_{t-1} = \ell$ (and hence $c_t = c_\ell$) is equal to $\Pr[s(\theta_t) = 1]$. Therefore, it suffices to prove that the illiberal government's period-t election probability when $g_{t-1} = i$ is equal across both equilibria. In particular, we prove this by showing that in both equilibria the illiberal government chooses the same censorship policy, i.e, $c^* = c'$. Following our argument in the Proof of Proposition 4 in Appendix A, in any III-III equilibrium, $\hat{\sigma}$, we have

$$A(\pi, q, L, S, \delta \mid \hat{\sigma}) = \frac{\hat{c}(0)\Pr[s(\theta_t) = 0]\Big(L - \Pr[\theta_t = 1 \mid s(\theta_t) = 0]S\Big)}{1 - \hat{c}(0)\delta\Pr[s(\theta_t) = 0]},$$

$\hat{c}(1) = 1$ and $\hat{c}(0)$ such that

$$\frac{\pi q + \hat{c}(0)\pi(1-q)}{\Pr[s(\theta_t) = 1] + \hat{c}(0)\Pr[s(\theta_t) = 0]}S = L + \delta A(\pi, q, L, S, \delta \mid \hat{\sigma}). \tag{10}$$

By assumption (σ^* and σ' exist), a solution $\hat{c}(0) \in [0, 1)$ exists. But then it must be unique because the left hand side of (10) is decreasing with $\hat{c}(0)$ and the right hand side is increasing with $\hat{c}(0)$. Therefore, $c^*(0) = c'(0)$ and, hence, $c^* = c'$. ■

Collectively, Lemmas 1′ and 2′ prove Lemma 5.